Trig – Equations + General Advice

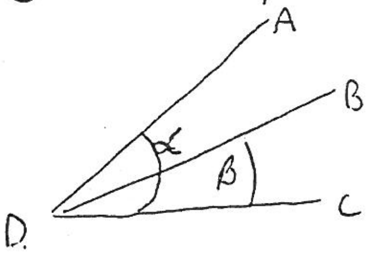
(1) Expect to require to use addition Formulae

(2) Expect to require to use $R\,{}^{\sin}_{\cos}(x \pm \alpha)$ (wave function)

(3) Watch for radians $\quad \sin x = 0.5$

No x° ∴ must use radians.

(4) Watch for sums / differences of angles.



Find Angle ADB or $\sin \angle ADB$

Angle ADB $= (\alpha - \beta)$

∴ $\sin(\alpha - \beta)$

Now use addition formulae

(5) Never do this!!!

$\cos(x+40)^\circ$

$= \cos x + \cos 40$

– this is rubbish!!

– use addition formulae.

$\cos(x+40) = \cos x \cos 40 - \sin x \sin 40$

(6) "Express in form $R\cos(12x - \alpha)$"

is often part of large question involving

Graph sketching

Solving equations (Type (3) inside →)

Interpreting Graphs etc

Look out for this question!!

(7) KNOW

Sine Rule

Cosine Rule

Area of △ Formulae !!!!!

Trig Equations

4 Types.

Type ① Single trig term.

Eg $\begin{cases} 5 + 4\cos 3x = 2 \\ 3\tan 4x - 3 = 1 \end{cases}$

Solve $5 + 4\cos 3x = 2$ $\quad 0 \leq x \leq 360°$

$\Rightarrow 4\cos 3x = -3$

$\Rightarrow \cos 3x = -0.75$

Let $A = 3x$

$\Rightarrow \cos A = -0.75$

Require 2^{nd} & 3^{rd} quadrant angle.

1^{st} quadrant Solve $\cos A_1 = 0.75$

$\Rightarrow A_1 = 41.4°$

2^{nd} quad $A_2 = 180° - 41.4° = 138.6°$

3^{rd} quad $A_3 = 180 + 41.4° = 221.4°$

$A = \begin{matrix} 138.6° \\ 221.4° \end{matrix} \Rightarrow 3x = \begin{matrix} 138.6° \\ 221.4° \end{matrix} \Rightarrow x = \begin{matrix} 46.2° \\ 73.8° \end{matrix}$

$x = \begin{matrix} 46.2° \\ 73.8° \end{matrix}$ are the FIRST CYCLE SOLUTIONS

$\cos 3x$ has 3 cycles to 360°

Period $= \frac{360}{3} = 120°$

First cycle		2nd cycle		3rd cycle
46.2	(+120)	166.2°	(+120)	286.2°
73.8	"	193.8°	"	313.8°

$x = 46.2°, 73.8°, 166.2°, 193.8°, 286.2°, 313.8°$

Type ② Two trig terms with different angle values ie $\sin 2x$ & $\cos x$
$\underbrace{\qquad\qquad}_{\text{different}}$

Technique
- Use double angle formulae to subst out "Double angle"
- All terms to L.S. = 0 on R.S.
- Factorise
  - if "$\sin 2x$" substituted likely to be common factor
  - if "$\cos 2x$" " " " trinomial
- Proceed with each factor as above in Type ①.
  (It is possible that one factor does not give solutions
  ie $(\sin x + 3) \Rightarrow \sin x \neq -3$
  Not possible

Examples over

② Ⓐ Subst for $\sin 2x$

$$\sin 2x = 3\sin x$$

* Formula Sheet $\sin 2x = 2\sin x\cos x$

$$2\sin x\cos x = 3\sin x$$

$$2\sin x\cos x - 3\sin x = 0$$

(sin x is common factor)

$$\sin x(2\cos x - 3) = 0$$

$\sin x = 0$ or $\cancel{\cos x = \frac{3}{2}}$ No solutions

↓

Proceed as type 1

2 Ⓑ Subst for $\cos 2x$

$$\cos 2x = 3\sin x - 1$$

$$\Rightarrow \cos 2x - 3\sin x + 1 = 0$$

* Formula sheet
$\cos 2x = \cos^2 x - \sin^2 x$
$\cos 2x = 2\cos^2 x - 1$
$\cos 2x = 1 - 2\sin^2 x$ ← Use this one!

$$\Rightarrow (1 - 2\sin^2 x) - 3\sin x + 1 = 0$$

$$\Rightarrow 1 - 2\sin^2 x - 3\sin x + 1 = 0$$

$$\Rightarrow 2\sin^2 x + 3\sin x - 2 = 0$$

trinomial $2s^2 + 3s - 2 =$
$(2s-1)(s+2) =$

$$\Rightarrow (2\sin x - 1)(\sin x + 2) = 0$$

$\Rightarrow \sin x = \frac{1}{2}$ or $\cancel{\sin x = -2}$ No solution

↓

Proceed as for type ①

* Both factors may give solutions.

Type ③ Two trig terms (sin & cos) with <u>same</u> angle value + another number term. ie $3\sin \underline{x}$, $4\cos \underline{x}$

Technique – Take two trig terms & express in form $k\cos(x \pm \alpha)$
or $k\sin(x \pm \alpha)$
(usually directed which one to use
– if not take your pick
but if 1st trig term is <u>cos</u> – use <u>cos</u> etc
makes algebra easier!

– Substitute this back into original equation

– Proceed as for type ① !!!

* This is often part of large Trig question – Probably 2nd paper

Type ③ Example.

Solve $3\cos 2x - 4\sin 2x = -1$

Let $3\cos 2x - 4\sin 2x = k\cos(2x - \alpha)$

$$= k(\cos 2x \cos\alpha + \sin 2x \sin\alpha)$$

$$= (k\cos\alpha)\underline{\cos 2x} + (k\sin\alpha)\underline{\sin 2x}$$

cf $\quad 3\cos 2x - 4\sin 2x$

$k\cos\alpha = 3$ $\quad k > 0 \quad$ $\cos\alpha$ +ve $\quad$ 1st or 4th $\quad$ $\therefore\ \alpha$ is 4th quad angle.

$k\sin\alpha = -4$ $\quad\quad$ $\sin\alpha$ −ve $\quad$ 3rd or 4th.

$$\frac{k\sin\alpha}{k\cos\alpha} = \frac{-4}{3} \Rightarrow \tan\alpha = -\frac{4}{3} \quad (4^{th}\text{ quad})$$

$$\alpha = 360 - \tan^{-1}\left(\frac{4}{3}\right)$$

$$\underline{\alpha = 306.8^\circ}$$

$k^2\cos^2\alpha = 9$

$k^2\sin^2\alpha = 16.$

$$\Rightarrow k^2(\underbrace{\cos^2\alpha + \sin^2\alpha}_{1}) = 25$$

$$\Rightarrow k = 5 \text{ since } k > 0$$

$$\therefore\ 3\cos 2x - 4\sin 2x = 5\cos(2x - 306.8)^\circ$$

Now solve equation $\quad 5\cos(2x - 306.8)^\circ = -1$

↓ Proceed as type ①.

Type ④ (Can be done as type ③ but much faster this way)
Two trig terms (sin & cos) with same angle value. but no other term (Non-zero term)

Ex $\quad$ Solve $\quad 5\sin x = 3\cos x$ ← Express in this form.

$$\Rightarrow \frac{5\sin x}{\cos x} = 3 \quad \text{Divide through by } \cos x. \quad \frac{\sin x}{\cos x} = \tan x$$

$$\Rightarrow 5\tan x = 3$$

↓ proceed as for type ①